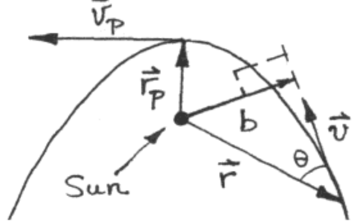
University of California, Berkeley
Physics 105 Fall 2000 Section 2 (*Strovink*)

ASSIGNMENT 2

Reading:
105 Notes 2.1, 2.2, 2.3, 2.4, 2.5.
Hand & Finch pp. 10-12, 130-134, 284-285.

1.
A comet, barely unbound by the sun (its total energy vanishes), executes a parabolic orbit about it.



At a certain time the comet has speed v and impact parameter b with respect to the sun. You may neglect the comet's mass m with respect to the sun's mass M. Find the perigee (distance of closest approach to the sun) of the comet.

2.
Two masses m_1 and m_2 orbit around their common center of mass (CM), which has the coordinate $\mathbf{R}(t)$. They are separated from the CM by $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$, respectively. Define

$$\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$
$$\mu \equiv \frac{m_1 m_2}{m_1 + m_2} \ ,$$

where μ is the *reduced mass*.
(**a**)
Show that the total kinetic energy $T = T_1 + T_2$ is equal to

$$T = \tfrac{1}{2}\mu\dot{\mathbf{r}}^2 + \tfrac{1}{2}M\dot{\mathbf{R}}^2 \ ,$$

where $M = m_1 + m_2$.
(**b**)
About the CM, show that the total angular momentum $\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2$ is equal to

$$\mathbf{L} = \mu\mathbf{r} \times \dot{\mathbf{r}} \ .$$

The simplicity of these formulæ explains why the two-particle separation $\mathbf{r}$ and the two-particle reduced mass μ are usually chosen as parameters for analysis of the two-body problem.

3.
Two particles connected by an elastic string of stiffness k and equilibrium length b rotate about their center of mass with angular momentum l. Show that their distances of closest and furthest approach, r_1 and r_2, are related by

$$r_1^2 r_2^2 (r_1 + r_2 - 2b) = (r_1 + r_2) l^2 / k\mu \ ,$$

where $\mu = m_1 m_2/(m_1 + m_2)$ is the two-body reduced mass.

4.
Determine which of the following forces are conservative, and find the potential energy (within a constant) for those which are:
(**a**)

$$F_x = 6abyz^3 - 20bx^3y^2$$
$$F_y = 6abxz^3 - 10bx^4y$$
$$F_z = 18abxyz^2 \ .$$

(**b**)

$$F_x = 18abyz^3 - 20bx^3y^2$$
$$F_y = 18abxz^3 - 10bx^4y$$
$$F_z = 6abxyz^2 \ .$$

(**c**)

$$\mathbf{F} = \hat{\mathbf{x}}F_1(x) + \hat{\mathbf{y}}F_2(y) + \hat{\mathbf{z}}F_3(z) \ .$$

5.
A vector field $\mathbf{F}$ is expressed in cylindrical coordinates as follows:

$$F_r = 0$$
$$F_\varphi = k/r$$
$$F_z = 0 \ ,$$

where k is a constant.

(**a**)

When $r > 0$, show that $\mathbf{F}$ has zero curl.

(**b**)

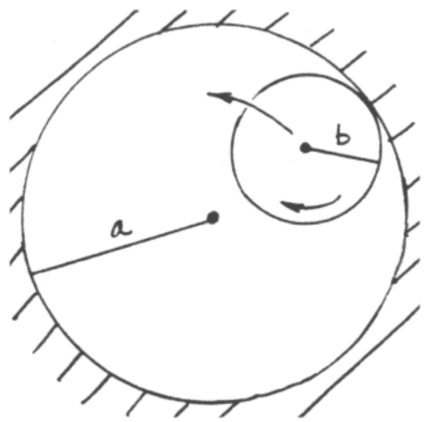
Consider the loop integral $\oint \mathbf{F} \cdot d\mathbf{l}$ counterclockwise around a circular path of fixed radius R. What is the value of this integral?

(**c**)

Can $\mathbf{F}$ be derived from a single-valued potential U, *e.g.* $\mathbf{F} = -\nabla U$ where $U = U(r, \varphi, z)$? Why or why not?

6.

A hoop of radius b and mass m rolls without slipping within a circular hole of radius $a > b$. About the center of the hole, the point of contact has a uniform angular velocity ω_a.



(**a**)

Find the angular velocity ω_b of the hoop about *its own* center (magnitude and direction).

(**b**)

Calculate the kinetic energy T of the hoop.

(**c**)

Obtain the angular momentum L (magnitude and direction) of the hoop relative to the center of the hole.

(**d**)

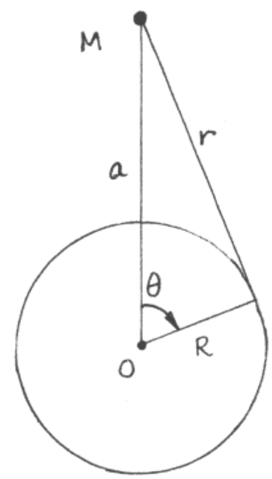
Consider your answers to (**b**) and (**c**) in the limit $b \to a$. You should find that both T and L vanish. Is this reasonable? Why or why not?

7.

Consider a spherically symmetric distribution $\rho(r)$ of mass density. If the gravitational acceleration, or "gravitational field vector" $\mathbf{g}$, is known to be independent of the radial coordinate r within a spherical volume, find $\rho(r)$ to within a multiplicative constant.

8.

Consider a point mass that lies outside a spherical surface. Let $\phi(\mathbf{r})$ be the gravitational potential due to the point mass.



Show that the average value of ϕ taken over the spherical surface is the same as the value of ϕ at the center of the sphere. [Since the potential due to an arbitrary mass distribution is the sum of potentials due to point masses, this statement is also true for the gravitational potential due to an arbitrary mass distribution lying outside a spherical surface.]